RATIONAL NUMBERS 7TH GRADE

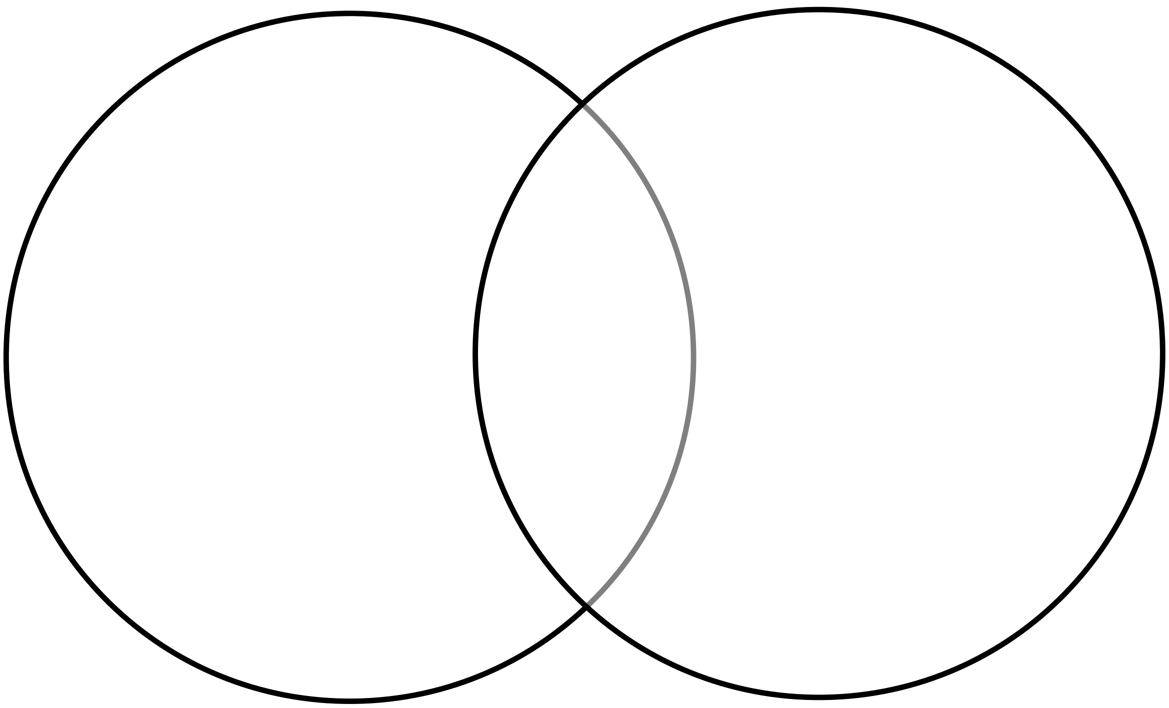
Lesson 1: Opposite Quantities Combine to Make Zero

Classwork

Exercise 1: Positive and Negative Numbers Review

With your partner, use the graphic organizer below to record what you know about positive and negative numbers. Add or remove statements during the whole class discussion.

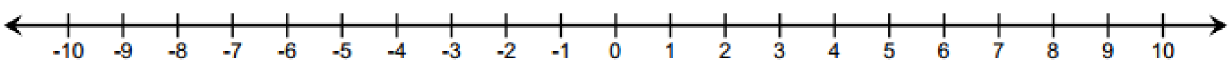
Negative Numbers **Positive Numbers**



Example 2: Counting Up and Counting Down on the Number Line

Use the number line below to practice counting up and counting down.

- *Counting up* corresponds to ______________________ numbers.
- *Counting down* corresponds to ______________________ numbers.



a. Where do you begin when locating a number on the number line?

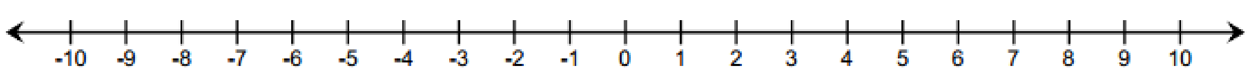
b. What do you call the distance between a number and 0 on a number line?

c. What is the relationship between 7 and -7?

Example 3: Using the Integer Game and the Number Line

What is the sum of the card values shown? Use the counting on method on the provided number line to justify your answer.

5	-5	-4	8

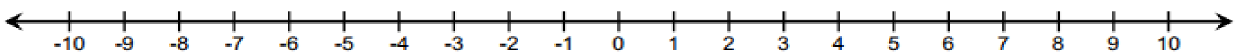


What is the final position on the number line? ________________________________

What card or combination of cards would you need to get back to 0? _____________________________

Exercise 2: The Additive Inverse

Use the number line to answer each of the following questions.



a. How far is 7 from 0 and in which direction? _______________________________

b. What is the opposite of 7? _______________________________

c. How far is -7 from 0 and in which direction? _______________________________

d. Thinking back to our previous work, how would you use the counting on method to represent the following: While playing the Integer Game, the first card selected is 7, and the second card selected is -7.

e. What does this tell us about the sum of 7 and its opposite, -7?

f. Look at the curved arrows you drew for 7 and -7. What relationship exists between these two arrows that would support your claim about the sum of 7 and -7?

g. Do you think this will hold true for the sum of any number and its opposite?

For all numbers a there is a number $-a$, such that $a + (-a) = 0$.

The additive inverse of a real number is the opposite of that number on the real number line. For example, the opposite of -3 is 3. A number and its additive inverse have a sum of 0. The sum of any number and its opposite is equal to zero.

Exercise 3: Playing the Integer Game

Play the Integer Game with your group. Use a number line to practice counting on.

Lesson 2: Using the Number Line to Model the Addition of Integers

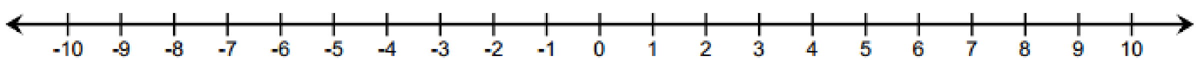
Classwork

Exercise 1: Real-World Introduction to Integer Addition

Answer the questions below.

a. Suppose you received $\$10$ from your grandmother for your birthday. You spent $\$4$ on snacks. Using addition, how would you write a number sentence to represent this situation?

b. How would you model your equation on a number line to show your answer?



Example 1: Modeling Addition on the Number Line

Complete the steps to finding the sum of $-2+3$ by filling in the blanks. Model the number sentence using straight arrows called *vectors* on the number line below.

a. Place the tail of the arrow on ________.

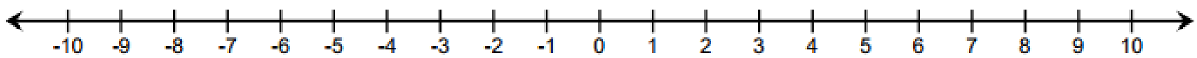
b. Draw the arrow 2 units to the left of 0, and stop at ________. The direction of the arrow is to the ________ since you are counting down from 0.

c. Start the next arrow at the end of the first arrow, or at ________.

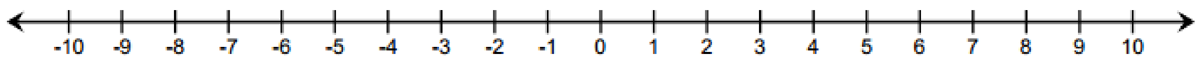
d. Draw the second arrow ________ units to the right since you are counting up from -2.

e. Stop at ________.

f. Circle the number at which the second arrow ends to indicate the ending value.



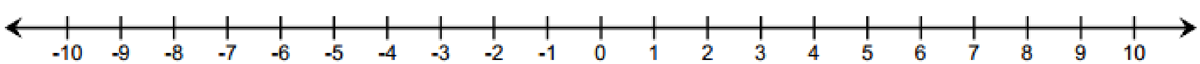
g. Repeat the process from parts (a)–(f) for the expression $3 + (-2)$.



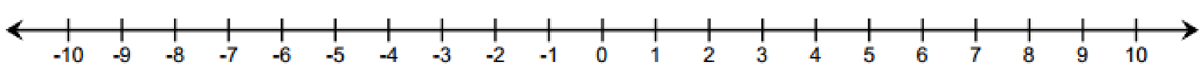
h. What can you say about the sum of $-2 + 3$ and $3 + (-2)$? Does order matter when adding numbers? Why or why not?

Example 2: Expressing Absolute Value as the Length of an Arrow on the Real Number Line

a. How does absolute value determine the arrow length for -2?



b. How does the absolute value determine the arrow length for 3?

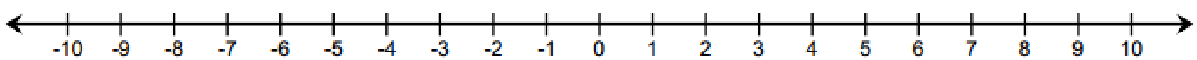


c. How does absolute value help you to represent -10 on a number line?

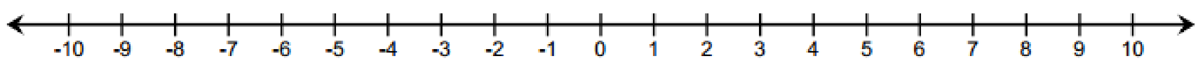
Exercise 2

Create a number line model to represent each of the expressions below.

a. $-6+4$

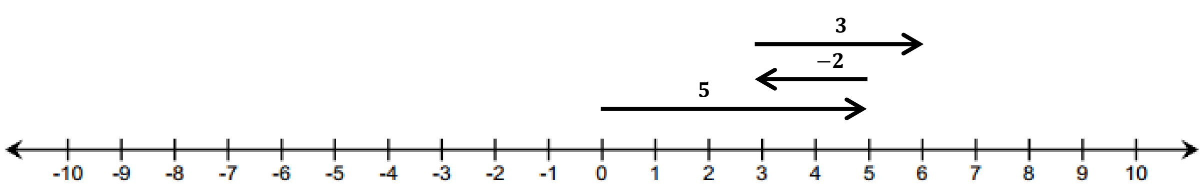


b. $3+(-8)$



Example 3: Finding Sums on a Real Number Line Model

Find the sum of the integers represented in the diagram below.



a. Write an equation to express the sum.

b. What three cards are represented in this model? How did you know?

c. In what ways does this model differ from the ones we used in Lesson 1?

d. Can you make a connection between the sum of 6 and where the third arrow ends on the number line?

e. Would the sum change if we changed the order in which we add the numbers, for example, $(-2) + 3 + 5$?

f. Would the diagram change? If so, how?

Exercise 3

Play the Integer Game with your group. Use a number line to practice "counting on".

Lesson 3: Understanding Addition of Integers

Classwork

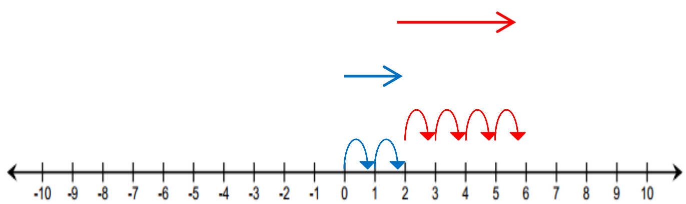
Exercise 1: Addition Using the Integer Game

Play the Integer Game with your group without using a number line.

Example 1: "Counting On" to Express the Sum as Absolute Value on a Number Line

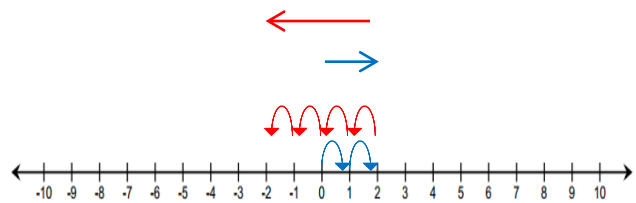
Model of Counting Up

$2 + 4 = 6$



Model of Counting Down

$2 + (-4) = -2$



Counting up -4 is the same as "the opposite of counting up 4" and also means counting down 4.

a. For each example above, what is the distance between 2 and the sum?

b. Does the sum lie to the right or left of 2 on a horizontal number line? Above or below on a vertical number line?

c. Given the expression $54 + 81$, determine, without finding the sum, the distance between 54 and the sum. Explain.

d. Is the sum to the right or left of 54 on the horizontal number line? Above or below on a vertical number line?

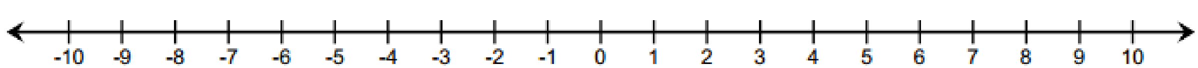
e. Given the expression $14 + (-3)$, determine, without finding the sum, the distance between 14 and the sum? Why?

f. Is the sum to the right or left of 14 on the number line? Above or below on a vertical number line?

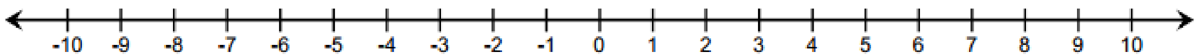
Exercise 2

Work with a partner to create a horizontal number line model to represent each of the following expressions. Describe the sum using distance from the p-value along the number line.

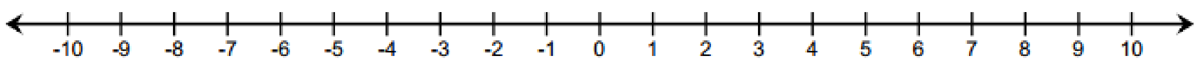
a. $-5 + 3$



b. $-6 + (-2)$



c. $7 + (-8)$

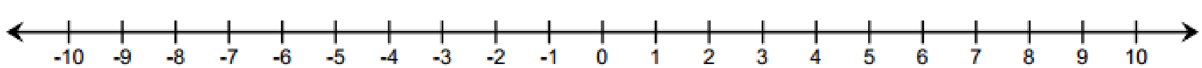


Exercise 3: Writing an Equation Using Verbal Descriptions

Write an equation, and using the number line, create an "arrow diagram" given the following information:

"The p-value is 6, and the sum lies 15 units to the left of the p-value."

Equation:

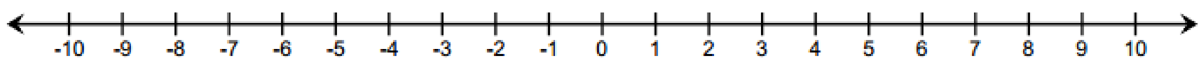


Lesson 4: Efficiently Adding Integers and Other Rational Numbers

Classwork

Example 1: Rule for Adding Integers with Same Signs

a. Represent the sum of $3 + 5$ using arrows on the number line.



i. How long is the arrow that represents 3?

ii. What direction does it point?

iii. How long is the arrow that represents 5?

iv. What direction does it point?

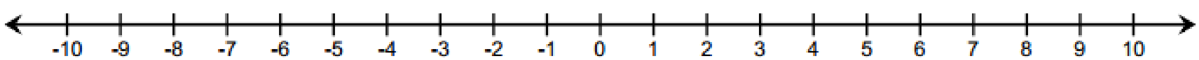
v. What is the sum?

vi. If you were to represent the sum using an arrow, how long would the arrow be and what direction would it point?

vii. What is the relationship between the arrow representing the number on the number line and the absolute value of the number?

viii. Do you think that adding two positive numbers will always give you a greater positive number? Why?

b. Represent the sum of $-3 + (-5)$ using arrows that represent -3 and -5 on the number line. From part (a), use the same questions to elicit feedback. In the Integer Game, I would combine -3 and -5 to give me -8.



i. How long is the arrow that represents -3?

ii. What direction does it point?

iii. How long is the arrow that represents -5?

iv. What direction does it point?

v. What is the sum?

vi. If you were to represent the sum using an arrow, how long would the arrow be and what direction would it point?

vii. Do you think that adding two negative numbers will always give you a smaller negative number? Why?

c. What do both examples have in common?

RULE: Add rational numbers with the same sign by adding the absolute values and using the common sign.

Exercise 2

Decide whether the sum will be positive or negative without actually calculating the sum.

i. $-4+(-2)$ ____________________

ii. $5+9$ ____________________

iii. $-6+(-3)$ ____________________

iv. $-1+(-11)$ ____________________

v. $3+5+7$ ____________________

vi. $-20+(-15)$ ____________________

Find the sum.

i. $15 + 7$

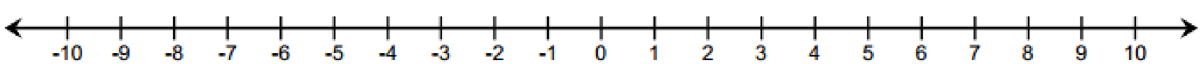
ii. $-4 + (-16)$

iii. $-18 + (-64)$

iv. $-205 + (-123)$

Example 2: Rule for Adding Opposite Signs

a. Represent the $5 + (-3)$ using arrows on the number line.



i. How long is the arrow that represents 5?

ii. What direction does it point?

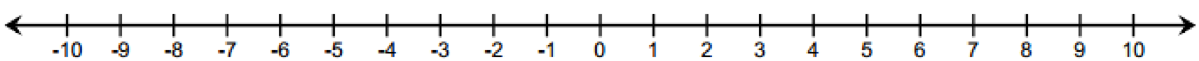
iii. How long is the arrow that represents -3?

iv. What direction does it point?

v. Which arrow is longer?

vi. What is the sum? If you were to represent the sum using an arrow, how long would the arrow be and what direction would it point?

b. Represent the $4 + (-7)$ using arrows on the number line.



i. In the two examples above, what is the relationship between length of the arrow representing the sum and the lengths of the arrows representing the p-value and q-value?

ii. What is the relationship between the direction of the arrow representing the sum and the direction of arrows representing the p-value and q-value?

iii. Write a rule that will give the length and direction of the arrow representing the sum of two values that have opposite signs.

RULE: Add rational numbers with opposite signs by subtracting the absolute values and using the sign of the integer with the greater absolute value.

Exercise 3

a. Circle the integer with the greater absolute value. Decide whether the sum will be positive or negative without actually calculating the sum.

i. $-1+2$

ii. $5+(-9)$

iii. $-6+3$

iv. $-11+1$

b. Find the sum.

i. $-10+7$

ii. $8+(-16)$

iii. $-12+65$

iv. $105+(-126)$

Example 3: Applying Integer Addition Rules to Rational Numbers

Find the sum of $6 + \left(-2\frac{1}{4}\right)$. The addition of rational numbers follows the same rules of addition for integers.

a. Find the absolute values of the numbers.

b. Subtract the absolute values.

c. The answer will take the sign of the number that has the greater absolute value.

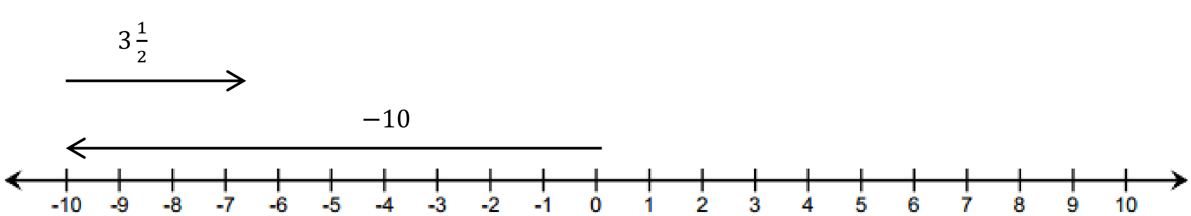
Exercise 4

Solve the following problems. Show your work.

a. Find the sum of $-18 + 7$.

b. If the temperature outside was 73 degrees at $5:00$ p.m., but it fell 19 degrees by 10:00 p.m., what is the temperature at 10:00 p.m.? Write an equation and solve.

c. Write an addition sentence, and find the sum using the diagram below.



Lesson 5: Understanding Subtraction of Integers and Other Rational Numbers

Classwork

Example 1: Exploring Subtraction with the Integer Game

Play the Integer Game in your group. Start Round 1 by selecting four cards. Follow the steps for each round of play.

1. Write the value of your hand in the Total column.
2. Then, record what card values you select in the Action 1 column and discard from your hand in the Action 2 column.
3. After each action, calculate your new total, and record it under the appropriate Results column.
4. Based on the results, describe what happens to the value of your hand under the appropriate Descriptions column. For example, "Score increased by 3."

Round	Total	Action 1	Result 1	Description	Action 2	Result 2	Description
1							
2							
3							
4							
5							

Discussion: Making Connections to Integer Subtraction

a. How did selecting positive value cards change the value of your hand?

b. How did selecting negative value cards change the value of your hand?

c. How did discarding positive value cards change the value of your hand?

d. How did discarding negative value cards change the value of your hand?

e. What operation reflects selecting a card?

f. What operation reflects discarding or removing a card?

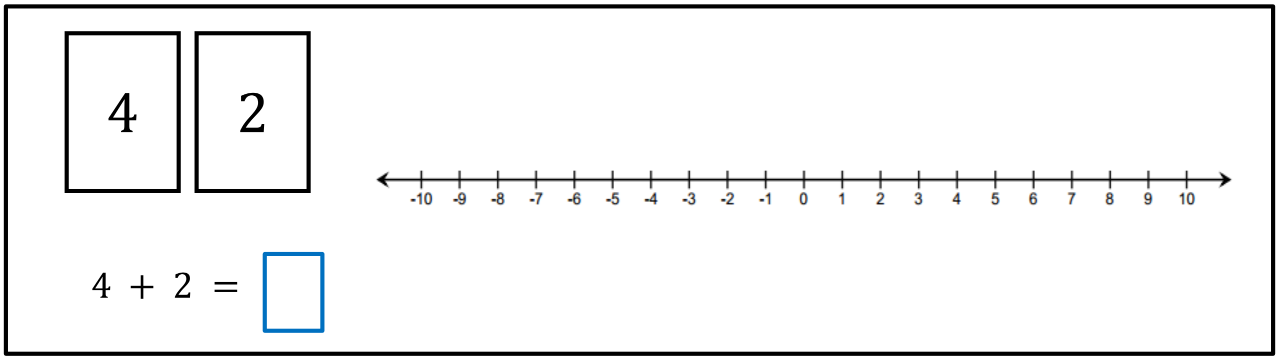
Based on the game, can you make a prediction about what happens to the result when

a. Subtracting a positive integer?

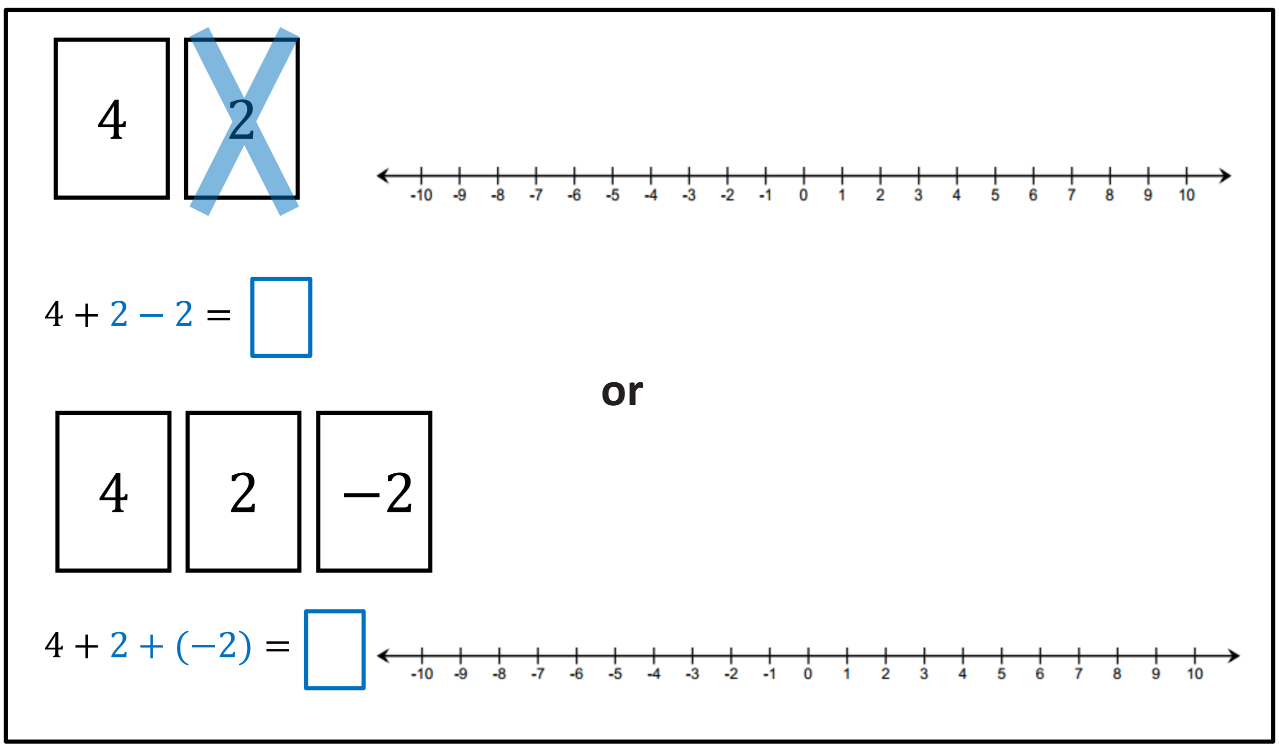
b. Subtracting a negative integer?

Example 2: Subtracting a Positive Number

Follow along with your teacher to complete the diagrams below.



Show that discarding (subtracting) a positive card, which is the same as subtracting a positive number, decreases the value of the hand.

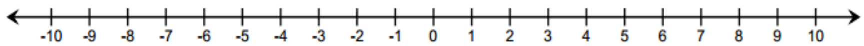


Removing (________________) a positive card changes the score in the same way as _____________ a card whose value is the ________________________ _______________________ (or opposite). In this case, adding the corresponding ____.

Example 3: Subtracting a Negative Number

Follow along with your teacher to complete the diagrams below.

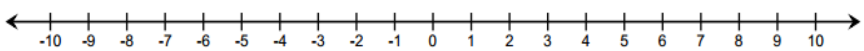
4 | -2



$4 + (-2) = \boxed{}$

How does removing a negative card change the score, or value, of the hand?

4 | $\cancel{-2}$

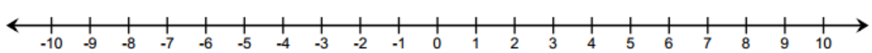


$4 + (-2) - (-2) = \boxed{}$

or

4 | -2 | 2

$4 + (-2) + 2 = \boxed{}$



Removing (________________) a negative card changes the score in the same way as _____________ a card whose value is the ________________________ _______________________ (or opposite). In this case, adding the corresponding ____.

The Rule of Subtraction: *Subtracting a number is the same as adding its additive inverse (or opposite).*

Exercises 1–3: Subtracting Positive and Negative Integers

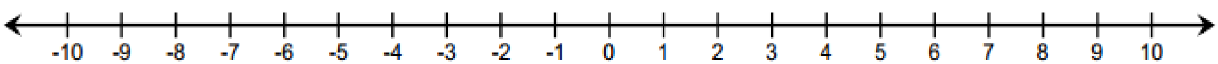
1. Using the rule of subtraction, rewrite the following subtraction sentences as addition sentences and solve. Use the number line below if needed.

   a. $8 - 2$

   b. $4 - 9$

   c. $-3 - 7$

   d. $-9 - (-2)$



2. Find the differences.

   a. $-2 - (-5)$

   b. $11 - (-8)$

   c. $10 - (-4)$

3. Write two equivalent expressions that represent the situation. What is the difference in their elevations?

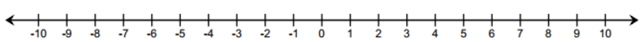
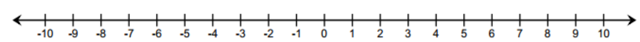
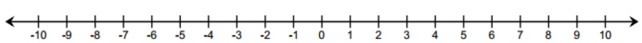
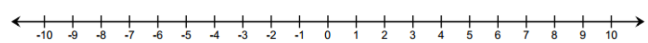
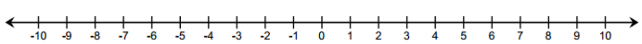
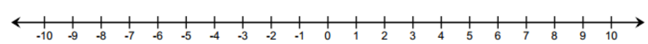
   "An airplane flies at an altitude of 25,000 feet. A submarine dives to a depth of 600 feet below sea level."

Lesson 6: The Distance Between Two Rational Numbers

Classwork

Exercise 1

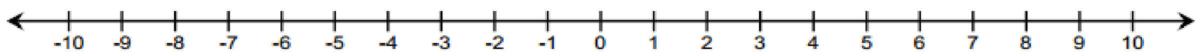
Use the number line to answer each of the following.

Person A	Person B
What is the distance between -4 and 5? 	What is the distance between 5 and -4? 
What is the distance between -5 and -3? 	What is the distance between -3 and -5? 
What is the distance between 7 and -1? 	What is the distance between -1 and 7? 

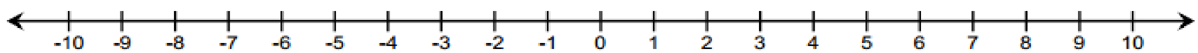
Exercise 2

Use the number line to answer each of the following questions.

a. What is the distance between 0 and -8?



b. What is the distance between -2 and $-1\frac{1}{2}$?



c. What is the distance between -6 and -10?

Example 1: Formula for the Distance Between Two Rational Numbers

Find the distance between –3 and 2.

Step 1: Start on an endpoint.

Step 2: Count the number of units from the endpoint you started on to the other endpoint.

Using a formula, ____

For two rational numbers p and q, the distance between p and q is $\lvert p - q \rvert$.

Example 2: Change in Elevation vs. Distance

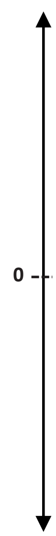
Distance is positive. Change in elevation or temperature may be positive or negative depending on whether it is increasing or decreasing (going up or down).

a. A hiker starts hiking at the beginning of a trail at a point which is 200 feet below sea level. He hikes to a location on the trail that is 580 feet above sea level and stops for lunch.

   i. What is the vertical distance between 200 feet below sea level and 580 feet above sea level?

   ii. How should we interpret 780 feet in the context of this problem?

b. After lunch, the hiker hiked back down the trail from the point of elevation, which is 580 feet above sea level, to the beginning of the trail which is 200 feet below sea level.



   i. What is the vertical distance between 580 feet above sea level and 200 feet below sea level?

   ii. What is the change in elevation?

Exercise 3

The distance between a negative number and a positive number is $12\frac{1}{2}$. What are the numbers?

Exercise 4

Use the distance formula to find each answer. Support your answer using a number line diagram.

a. Find the distance between -7 and -4.

b. Find the change in temperature if the temperature rises from $-18°\mathrm{F}$ to $15°\mathrm{F}$ (use a vertical number line).

c. Would your answer for part (b) be different if the temperature dropped from $15°\mathrm{F}$ to $-18°\mathrm{F}$? Explain.

d. Beryl is the first person to finish a 5K race and is standing 15 feet beyond the finish line. Another runner, Jeremy, is currently trying to finish the race and has approximately 14 feet before he reaches the finish line. What is the minimum possible distance between Beryl and Jeremy?

e. What is the change in elevation from 140 feet above sea level to 40 feet below sea level? Explain.

Lesson 7: Addition and Subtraction of Rational Numbers

Classwork

Exercise 1: Real-World Connection to Adding and Subtracting Rational Numbers

Suppose a 7th grader's birthday is today, and she is 12 years old. How old was she $3\frac{1}{2}$ years ago? Write an equation and use a number line to model your answer.

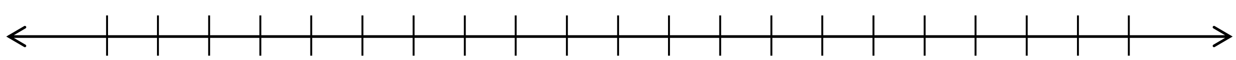
Example 1: Representing Sums of Rational Numbers on a Number Line

a. Place the tail of the arrow on 12.

b. The length of the arrow is the absolute value of $-3\frac{1}{2}$, $\left|-3\frac{1}{2}\right| = 3\frac{1}{2}$.

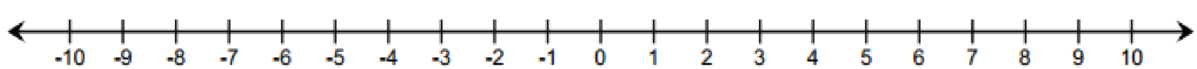
c. The direction of the arrow is to the *left* since you are adding a negative number to 12.

Draw the number line model in the space below.



Exercise 2

Find the following sum using a number line diagram. $-2\frac{1}{2} + 5$.



Example 2: Representing Differences of Rational Numbers on a Number Line

a. Rewrite the difference $1 - 2\frac{1}{4}$ as a sum: $1 + \left(-2\frac{1}{4}\right)$.

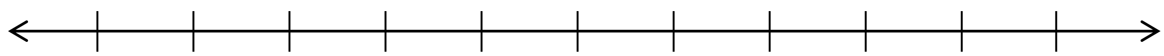
<u>Now follow the steps to represent the sum</u>:

b. Place the tail of the arrow on 1.

c. The length of the arrow is the absolute value of $-2\frac{1}{4}$; $\left|-2\frac{1}{4}\right| = 2\frac{1}{4}$.

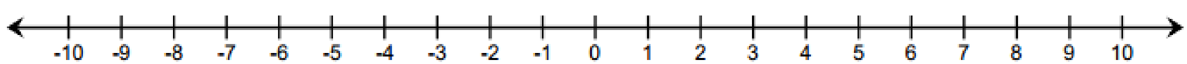
d. The direction of the arrow is to the *left* since you are adding a negative number to 1.

Draw the number line model in the space below.



Exercise 3

Find the following difference, and represent it on a number line. $-5\frac{1}{2} - (-8)$.



Exercise 4

Find the following sums and differences using a number line model.

a. $-6 + 5\frac{1}{4}$

b. $7 - (-0.9)$

c. $2.5 + \left(-\frac{1}{2}\right)$

d. $-\frac{1}{4} + 4$

e. $\frac{1}{2} - (-3)$

Exercise 5

Create an equation and number line diagram to model each answer.

a. Samantha owes her father $\$7$. She just got paid $\$5.50$ for babysitting. If she gives that money to her dad, how much will she still owe him?

b. At the start of a trip, a car's gas tank contains 12 gallons of gasoline. During the trip, the car consumes $10\frac{1}{8}$ gallons of gasoline. How much gasoline is left in the tank?

c. A fish was swimming $3\frac{1}{2}$ feet below the water's surface at $7{:}\,00$ a.m. Four hours later, the fish was at a depth that is $5\frac{1}{4}$ feet below where it was at $7{:}\,00$ a.m. What rational number represents the position of the fish with respect to the water's surface at $11{:}\,00$ a.m.?

Lesson 8: Applying the Properties of Operations to Add and Subtract Rational Numbers

Classwork

Example 1: The Opposite of a Sum is the Sum of its Opposites

Explain the meaning of: "The opposite of a sum is the sum of its opposites." Use a specific math example.

Rational Number	Rational Number	Sum	Opposite of the Sum

Opposite Rational Number	Opposite Rational Number	Sum

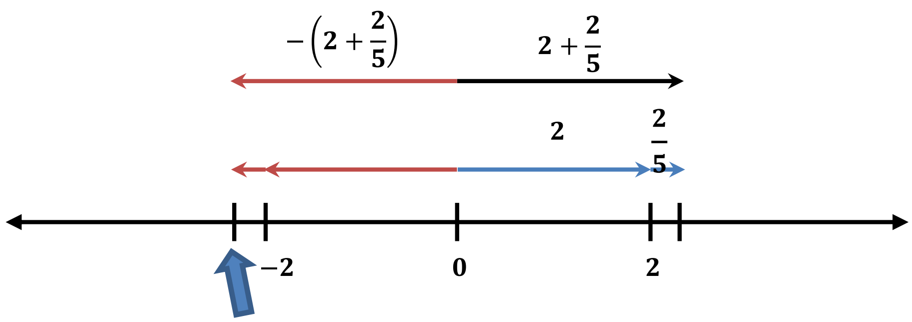
Exercise 1

Represent the following expression with a single rational number.

$$-2\frac{2}{5}+3\frac{1}{5}-\frac{3}{5}$$

Example 2: A Mixed Number is a Sum

Use the number line model shown below to explain and write the opposite of $2\frac{2}{5}$ as a sum of two rational numbers.



The opposite of a sum (top single arrow pointing left) and the sum of the opposites correspond to the same point on the number line.

Exercise 2

Rewrite each mixed number as the sum of two signed numbers.

a. $-9\frac{5}{8}$

b. $-2\frac{1}{2}$

c. $8\frac{11}{12}$

Exercise 3

Represent each sum as a mixed number.

a. $-1+\left(-\frac{5}{12}\right)$

b. $30+\frac{1}{8}$

c. $-17+\left(-\frac{1}{9}\right)$

Exercise 4

Mr. Mitchell lost 10 pounds over the summer by jogging each week. By winter time, he had gained $5\frac{1}{8}$ pounds. Represent this situation with an expression involving signed numbers. What is the overall change in Mr. Mitchell's weight?

Exercise 5

Jamal is completing a math problem and represents the expression $-5\frac{5}{7}+8-3\frac{2}{7}$ with a single rational number as shown in the steps below. Justify each of Jamal's steps. Then, show another way to solve the problem.

$= -5\frac{5}{7} + 8 + \left(-3\frac{2}{7}\right)$

$= -5\frac{5}{7} + \left(-3\frac{2}{7}\right) + 8$

$= -5 + \left(-\frac{5}{7}\right) + (-3) + \left(-\frac{2}{7}\right) + 8$

$= -5 + \left(-\frac{5}{7}\right) + \left(-\frac{2}{7}\right) + (-3) + 8$

$= -5 + (-1) + (-3) + 8$

$= -6 + (-3) + 8$

$= (-9) + 8$

$= -1$

Lesson 9: Applying the Properties of Operations to Add and Subtract Rational Numbers

Classwork

Exercise 1

Unscramble the cards, and show the steps in the correct order to arrive at the solution to $5\frac{2}{9} - \left(8.1 + 5\frac{2}{9}\right)$.

$0 + (-8.1)$
$\left(5\frac{2}{9} + \left(-5\frac{2}{9}\right)\right) + (-8.1)$
-8.1
$5\frac{2}{9} + \left(-8.1 + \left(-5\frac{2}{9}\right)\right)$
$5\frac{2}{9} + \left(-5\frac{2}{9} + (-8.1)\right)$

Examples 1–2

Represent each of the following expressions as one rational number. Show and explain your steps.

a. $4\frac{4}{7} - (4\frac{4}{7} - 10)$

b. $5 + \left(-4\frac{4}{7}\right)$

Exercise 2: Team Work!

a. $-5.2 - (-3.1) + 5.2$

b. $32 + \left(-12\frac{7}{8}\right)$

c. $3\frac{1}{6} + 20.3 - \left(-5\frac{5}{6}\right)$

d. $\frac{16}{20} - (-1.8) - \frac{4}{5}$

Exercise 3

Explain step by step, how to arrive at a single rational number to represent the following expression. Show both a written explanation and the related math work for each step.

$$-24 - \left(-\frac{1}{2}\right) - 12.5$$

Lesson 10: Understanding Multiplication of Integers

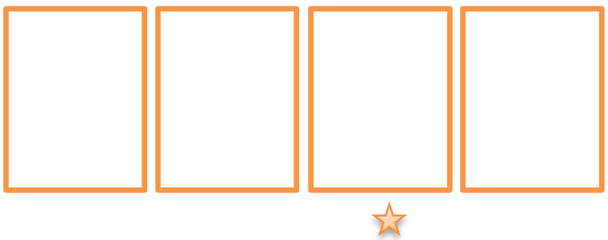
Classwork

Exercise 1: Integer Game Revisited

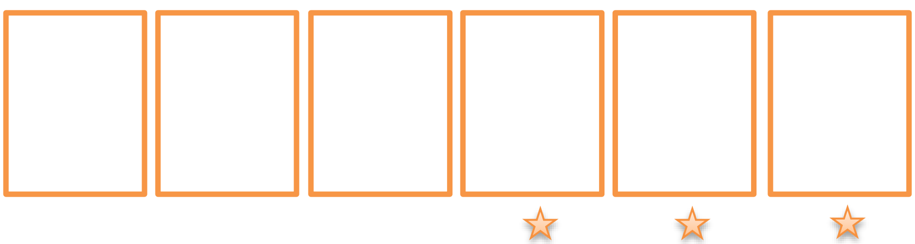
In groups of four, play one round of the Integer Game (see Integer Game outline for directions).

Example 1: Product of a Positive Integer and a Negative Integer

Part A:



Part B:



Use your cards from Part B to answer the questions below.

a. Write a product that describes the three matching cards.

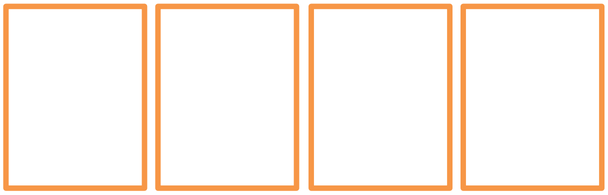
b. Write an expression that represents how each of the ✯ cards changes your score.

c. Write an equation that relates these two expressions.

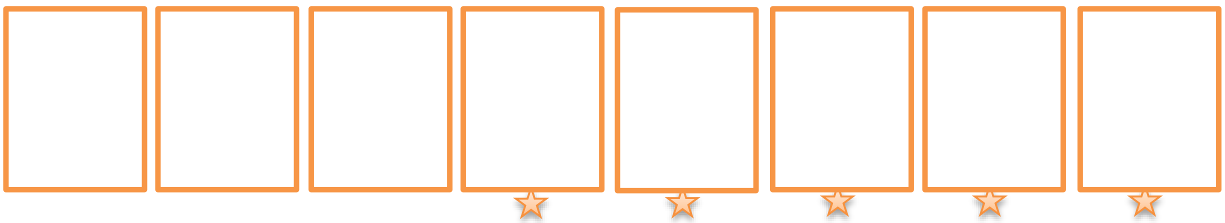
d. Write an integer that represents the total change to your score by the three ✯ cards.

e. Write an equation that relates the product and how it affects your score.

Part C:



Part D:



Use your cards from Part D to answer the questions below.

f. Write a product that describes the five matching cards.

g. Write an expression that represents how each of the ✯ cards changes your score.

h. Write an equation that relates these two expressions.

i. Write an integer that represents the total change to your score by the three ✯ cards.

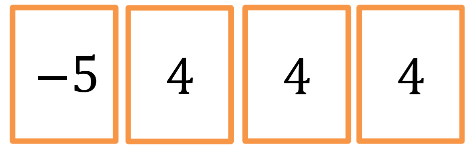
j. Write an equation that relates the product and how it affects your score.

k. Use the expression 5×4 to relate the multiplication of a positive valued card to addition.

l. Use the expression $3 \times (-5)$ to relate the multiplication of a negative valued card to addition.

Example 2: Product of a Negative Integer and a Positive Integer

a. If all of the 4's from the playing hand on the right are discarded, how will the score be affected? Model this using a product in an equation.

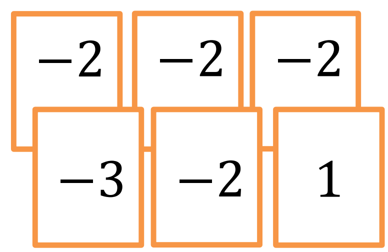


b. What three matching cards could be added to those pictured to get the same change in score? Model this using a product in an equation.

c. Seeing how each play affects the score, relate the products that you used to model them. What do you conclude about multiplying integers with opposite signs?

Example 3: Product of Two Negative Integers

a. If the matching cards from the playing hand on the right are discarded, how will this hand's score be affected? Model this using a product in an equation.



b. What four matching cards could be added to those pictured to get the same change in score? Model this using a product in an equation.

c. Seeing how each play affects the score, relate the products that you used to model them. What do you conclude about multiplying integers with the same sign?

d. Using the conclusions from Examples 2 and 3, what can we conclude about multiplying integers? Write a few examples.

Lesson 11: Develop Rules for Multiplying Signed Numbers

Classwork

Example 1: Extending Whole Number Multiplication to the Integers

Part A: Complete quadrants I and IV of the table below to show how sets of matching integer cards will affect a player's score in the Integer Game. For example, three 2's would increase a player's score by $0 + 2 + 2 + 2 = 6$ points.

Quadrant II Quadrant I

What does this quadrant represent? (Quadrant II)

____
____
____
____

What does this quadrant represent? (Quadrant I)

____
____
____
____

					5					
					4					
					3					
					2			6		
					1					
						1	2	3	4	5
					-1					
					-2					
					-3					
					-4					
					-5					

Number of matching cards

Quadrant III Quadrant IV

Integer card values

What does this quadrant represent? (Quadrant III)

____
____
____
____

What does this quadrant represent? (Quadrant IV)

____
____
____
____

a. What patterns do you see in the right half of the table?

b. Enter the missing integers in the left side of the middle row and describe what they represent.

Part B: Complete quadrant II of the table.

c. What relationships or patterns do you notice between the produtcs (values) in quadrant II and the products (values) in quadrant I?

d. What relationships or patterns do you notice between the products (values) in quadrant II and the products (values) in quadrant IV?

e. Use what you know about the products (values) in quadrants I, II, and IV to describe what quadrant III will look like when its products (values) are entered.

Part C: Complete the quadrant III of the table.

Refer to the completed table to help you answer the following questions:

f. Is it possible to know the sign of a product of two integers just by knowing in which quadrant each integer is located? Explain.

g. Which quadrants contain which values? Describe an integer game scenario represented in each quadrant.

Exercise 1: Multiplication of Integers in the Real World

Generate real-world situations that can be modeled by each of the following multiplication problems. Use the Integer Game as a resource.

a. -3×5

b. $-6 \times (-3)$

c. $4 \times (-7)$

Lesson 12: Division of Integers

Classwork

Exercise 1: Recalling the Relationship Between Multiplication and Division

Record equations from Exercise 1 on the left.

Equations **Integers**

Example 1: Transitioning from Integer Multiplication Rules to Integer Division Rules

Record your group's number sentences in the space on the left below.

Equations **Integers**

a. List examples of division problems that produced a quotient that is a negative number.

b. If the quotient is a negative number, what must be true about the signs of the dividend and divisor?

c. List your examples of division problems that produced a quotient that is a positive number.

d. If the quotient is a positive number, what must be true about the signs of the dividend and divisor?

Rules for Dividing Two Integers:

- A quotient is negative if the divisor and the dividend have ________________________ signs.
- A quotient is positive if the divisor and the dividend have ________________________ signs.

Exercise 2: Is the Quotient of Two Integers Always an Integer?

Is the quotient of two integers always an integer? Use the work space below to create quotients of integers. Answer the question and use examples or a counterexample to support your claim.

Work Space:

Answer:

Exercise 3: Different Representation of the Same Quotient

Are the answers to the three quotients below the same or different? Why or why not?

a. $-14 \div 7$

b. $14 \div (-7)$

c. $-(14 \div 7)$

Lesson 13: Converting Between Fractions and Decimals Using Equivalent Fractions

Classwork

Example 1: Representations of Rational Numbers in the Real World

Following the Opening Exercise and class discussion, describe why we need to know how to represent rational numbers in different ways.

Example 2: Using Place Values to Write (Terminating) Decimals as Equivalent Fractions

a. What is the value of the number 2.25? How can this number be written as a fraction or mixed number?

b. Rewrite the fraction in its simplest form showing all steps that you use.

c. What is the value of the number 2.025? How can this number be written as a mixed number?

d. Rewrite the fraction in its simplest form showing all steps that you use.

Exercise 1

Use place value to convert each terminating decimal to a fraction. Then rewrite each fraction in its simplest form.

a. 0.218

b. 0.16

c. 2.72

d. 0.0005

Example 3: Converting Fractions to Decimals—Fractions with Denominators Having Factors of only 2 or 5

a. What are *decimals*?

b. Use the meaning of *decimal* to relate decimal place values.

c. Write the number $\frac{3}{100}$ as a decimal. Describe your process.

d. Write the number $\frac{3}{20}$ as a decimal. Describe your process.

e. Write the number $\frac{10}{25}$ as a decimal. Describe your process.

f. Write the number $\frac{8}{40}$ as a decimal. Describe your process.

Exercise 2

Convert each fraction to a decimal using an equivalent fraction.

a. $\frac{3}{16} =$

b. $\frac{7}{5} =$

c. $\frac{11}{32} =$

d. $\frac{35}{50} =$

Lesson 14: Converting Rational Numbers to Decimals Using Long Division

Classwork

Example 1: Can All Rational Numbers Be Written as Decimals?

a. Using the division button on your calculator, explore various quotients of integers 1 through 11. Record your fraction representations and their corresponding decimal representations in the space below.

b. What two types of decimals do you see?

Example 2: Decimal Representations of Rational Numbers

In the chart below, organize the fractions and their corresponding decimal representation listed in Example 1 according to their type of decimal.

What do these fractions have in common?	What do these fractions have in common?

Example 3: Converting Rational Numbers to Decimals Using Long Division

Use the long division algorithm to find the decimal value of $-\frac{3}{4}$.

Exercise 1

Students convert each rational number to its decimal form using long division.

a. $-\frac{7}{8} =$

b. $\frac{3}{16} =$

Example 4: Converting Rational Numbers to Decimals Using Long Division

Use long division to find the decimal representation of $\frac{1}{3}$.

Exercise 2

Calculate the decimal values of the fraction below using long division. Express your answers using bars over the shortest sequence of repeating digits.

a. $-\frac{4}{9}$

b. $-\frac{1}{11}$

c. $\frac{1}{7}$

d. $-\frac{5}{6}$

Example 5: Fractions Represent Terminating or Repeating Decimals

How do we determine whether the decimal representation of a quotient of two integers, with the divisor not equal to zero, will terminate or repeat?

Example 6: Using Rational Number Conversions in Problem Solving

a. Eric and four of his friends are taking a trip across the New York State Thruway. They decide to split the cost of tolls equally. If the total cost of tolls is $\$8$, how much will each person have to pay?

b. Just before leaving on the trip, two of Eric's friends have a family emergency and cannot go. What is each person's share of the $\$8$ tolls now?

Lesson 15: Multiplication and Division of Rational Numbers

Classwork

Exercise 1

a. In the space below, create a word problem that involves integer multiplication. Write an equation to model the situation.

b. Now change the word problem by replacing the integers with non-integer rational numbers (fractions or decimals), and write the new equation.

c. Was the process used to solve the second problem different from the process used to solve the first? Explain.

d. The Rules for Multiplying Rational Numbers are the same as the Rules for Multiplying Integers:

1. ____

2. ____

3. ____

Exercise 2

a. In one year, Melinda's parents spend $\$2{,}640.90$ on cable and internet service. If they spend the same amount each month, what is the resulting monthly change in the family's income?

b. The Rules for Dividing Rational Numbers are the same as the Rules for Dividing Integers:

1. ____

2. ____

3. ____

Exercise 3

Use the fundraiser chart to help answer the questions that follow.

Grimes Middle School Flower Fundraiser

Customer	**Plant Type**	**Number of Plants**	**Price per Plant**	**Total**	**Paid? Yes or No**
Tamara Jones	tulip	2	$4.25		No
Mrs. Wolff	daisy	1	$3.75	$ 3.75	Yes
Mr. Clark	geranium	5	$2.25		Yes
Susie (Jeremy's sister)	violet	1	$2.50	$ 2.50	Yes
Nana and Pop (Jeremy's grandparents)	daisy	4	$3.75	$15.00	No

Jeremy is selling plants for the school's fundraiser, and listed on the previous page is a chart from his fundraiser order form. Use the information in the chart to answer the following questions. Show your work and represent the answer as a rational number; then, explain your answer in the context of the situation.

a. If Tamara Jones writes a check to pay for the plants, what is the resulting change in her checking account balance?

Numerical Answer:

Explanation:

b. Mr. Clark wants to pay for his order with a $20 bill, but Jeremy does not have change. Jeremy tells Mr. Clark he will give him the change later. How will this affect the total amount of money Jeremy collects? Explain. What rational number represents the change that must be made to the money Jeremy collects?

Numerical Answer:

Explanation:

c. Jeremy's sister, Susie, borrowed the money from their mom to pay for her order. Their mother has agreed to deduct an equal amount of money from Susie's allowance each week for the next five weeks to repay the loan. What is the weekly change in Susie's allowance?

Numerical Answer:

Explanation:

d. Jeremy's grandparents want to change their order. They want to order three daisies and one geranium, instead of four daisies. How does this change affect the amount of their order? Explain how you arrived at your answer.

e. Jeremy approaches three people who do not want to buy any plants; however, they wish to donate some money for the fundraiser when Jeremy delivers the plants one week later. If the people promise to donate a total of $\$14.40$, what will be the average cash donation?

f. Jeremy spends one week collecting orders. If 22 people purchase plants totaling $\$270$, what is the average amount of Jeremy's sale?

Lesson 16: Applying the Properties of Operations to Multiply and Divide Rational Numbers

Classwork

Example 1: Using the Commutative and Associative Properties to Efficiently Multiply Rational Numbers

a. Evaluate the expression below.

$$-6 \times 2 \times (-2) \times (-5) \times (-3)$$

b. What types of strategies were used to evaluate the expressions?

c. Can you identify the benefits of choosing one strategy versus another?

d. What is the sign of the product and how was the sign determined?

Exercise 1

Find an efficient strategy to evaluate the expression and complete the necessary work.

$$-1 \times (-3) \times 10 \times (-2) \times 2$$

Exercise 2

Find an efficient strategy to evaluate the expression and complete the necessary work.

$$4 \times \frac{1}{3} \times (-8) \times 9 \times \left(-\frac{1}{2}\right)$$

Exercise 3

What terms did you combine first and why?

Exercise 4

Refer to the example and exercises. Do you see an easy way to determine the sign of the product first?

Example 2: Using the Distributive Property to Multiply Rational Numbers

Rewrite the mixed number as a sum; then, multiply using the distributive property.

$-6 \times \left(5\frac{1}{3}\right)$

Exercise 5

Multiply the expression using the distributive property.

$9 \times \left(-3\frac{1}{2}\right)$

Example 3: Using the Distributive Property to Multiply Rational Numbers

Evaluate using the distributive property.

$$16 \times \left(-\frac{3}{8}\right) + 16 \times \frac{1}{4}$$

Example 4: Using the Multiplicative Inverse to Rewrite Division as Multiplication

Rewrite the expression as only multiplication and evaluate.

$$1 \div \frac{2}{3} \times (-8) \times 3 \div \left(-\frac{1}{2}\right)$$

Exercise 6

$$4.2 \times \left(-\frac{1}{3}\right) \div \frac{1}{6} \times (-10)$$